

### AS-490

## M.Sc. (Maths) IV Semester (Reg./Pvt./ATKT) Examination June 2019

### INTEGRAL TRANSFORM - II

Optional (any four)

Paper - X

Time Allowed : Three Hours] [Maximum Marks :  $\begin{cases} \text{Reg.-85} \\ \text{Pvt.-100} \end{cases}$

Note : Attempt all questions.

Section - A

Objective Type Questions

15 × 1 = 15

Q.1. Choose the correct answer:

i) Fourier sine transform of  $\frac{e^{-ax}}{x}$  is

(a)  $\frac{a}{a^2 + s^2}$

(b)  $\frac{s}{a^2 + s^2}$

~~(c)~~  $\tan^{-1} \frac{s}{a}$

(d) None of these

ii) The complex fourier transform of  $e^{-|x|}$  is -

(a)  $\frac{1}{1+s^2}$

(b)  $\frac{1}{1-s^2}$

(c)  $\frac{2}{1-s^2}$

~~(d)~~  $\frac{2}{1+s^2}$

iii) If  $F\{f(x)\} = \tilde{f}(p)$ , then  $F\{f(ax)\}$  is equal to

(a)  $\frac{1}{a} \tilde{f}(ap)$

~~(b)~~  $\frac{1}{a} \tilde{f}(p/a)$

(c)  $\frac{1}{a} \tilde{f}(a/p)$

(d)  $a \tilde{f}(p/a)$

iv) The finite fourier sine transform of  $f(x) = \sin nx$ , where  $n$  is a positive integer and  $p \neq n$  is.

(a)  $\frac{\pi}{2}$

~~(b)~~  $\frac{\pi p}{2}$

(c) 0

(d)  $\pi p$

v) The fourier transform of

$$f(x) = \begin{cases} 1-x^2, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases} \text{ is}$$

- ~~(a)~~  $e^x$                       (b)  $e^{-x}$   
 (c) 1                              (d) None of these

vi) If  $F\{f(x)\} = \tilde{f}(s)$ , then  $F\left\{\frac{d^n f}{dx^n}\right\} = \dots\dots$

- (a)  $s^n \tilde{f}(s)$               ~~(b)~~  $s^{n-1} \tilde{f}(s)$   
 (c)  $s^n \tilde{f}^n(s)$               (d) None of these

vii) The series,  $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{l}$  is known as

- ~~(a)~~ Fourier series  
 (b) Fourier sine series  
 (c) Fourier cosine series  
 (d) Euler's series

viii) If  $F(s)$  is the complex Fourier transform of  $f(x)$ , then  $F\{f(x) \cos ax\}$  is -

- ~~(a)~~  $\frac{1}{2}[F(s+a) + F(s-a)]$   
 (b)  $\frac{1}{2}[F(s+a) - F(s-a)]$   
 (c)  $\frac{1}{2}[F(s+a) \cdot F(s-a)]$   
 (d)  $\frac{1}{2}[F(s+a) / F(s-a)]$

ix) Which is the correct relations

- ~~(a)~~  $\frac{2}{\pi} \int_0^{\infty} f_c(p) \cdot g_c(p) dp = \int_0^{\infty} F(x)G(x) dx$   
 (b)  $\frac{2}{\pi} \int_0^{\infty} f_s(p) \cdot g_s(p) dp = \int_0^{\infty} F(x)G(x) dx$   
 (c)  $\frac{2}{\pi} \int_0^{\infty} |f_c(p)|^2 dp = \int_0^{\infty} |F(x)|^2 dx$   
 (d) All of the above

x) If  $\phi(p)$  is the fourier sine transform of  $f(x)$  for  $p > 0$ , then  $F_s\{f(x)\}$  is equal to

- (a)  $-\phi(p), p < 0$  (b)  $-\phi(-p), p < 0$   
 (c)  $-\phi(-p), p > 0$  (d)  $\phi(-p), p < 0$

xi) The cosine transform of  $e^{-x}$  is

- ~~(a)~~  $\frac{s}{1+s^2}$  (b)  $\frac{1}{1+s^2}$   
 (c)  $\frac{1}{1-s^2}$  (d)  $\frac{s}{1-s^2}$

xii) The Hankel transform of  $\frac{e^{-x}}{x}$ , as taking  $xJ_0(px)$  as the kernel of the transformation

- ~~(a)~~  $\frac{1}{\sqrt{1+p^2}}$  (b)  $\frac{1}{(1+p^2)^{1/2}}$   
 (c)  $\frac{p}{(1+p^2)^{1/2}}$  (d) None of these

xiii) The Hankel transform of  $\frac{\cos ax}{x}$ , on taking kernel of transform,  $xJ_0(px)$  ( $p < a$ ) is

- (a)  $(p^2 - a^2)^{1/2}$  (b)  $(p^2 - a^2)^{3/2}$   
~~(c)~~  $\frac{p}{(p^2 + a^2)^{1/2}}$  (d) None of these

xiv)  $M\{f(ax)\} = \underline{\hspace{2cm}}$

- (a)  $\frac{1}{a} \& \left(\frac{p}{a}\right)$  (b)  $\frac{1}{ap} \overline{f(p)}$   
~~(c)~~  $a f\left(\frac{p}{a}\right)$  (d) None of these

xv)  $M(e^{-x}) = \underline{\hspace{2cm}}$

- ~~(a)~~  $\sqrt{p} (p < 0)$  (b)  $\frac{1}{2} p (p < 0)$   
 (c)  $\frac{p}{2}$  (d)  $\frac{1}{2} e^{p^2}$

Section - B

Short Answer Type Questions

5 × 5 = 25

Q.2. Find the fourier sine transform of

$$f(x) = \frac{1}{e^{\pi x} - e^{-\pi x}}$$

OR

Find sine transform of  $\frac{e^{-ax}}{x}$

Q.3. Find the finite cosine transform of  $f(x)$ , if

$$f(x) = \frac{\pi}{3} - x + \frac{x^2}{2\pi}$$

OR

Find the finite cosine transform of  $f(x)$ , if

$$f(x) = \begin{cases} 1, & 0 < x < \pi/2 \\ -1, & \pi/2 \leq x < \pi \end{cases}$$

Q.4. Using fourier integral, show that

$$e^{-ax} = \frac{2a}{\pi} \int_0^{\infty} \frac{\cos \lambda x}{\lambda^2 + a^2} d\lambda, \quad a > 0, x \geq 0$$

OR

By using fourier integral formula, show that

$$f(x) = \frac{2}{\pi} \int_0^{\infty} \int_0^{\infty} f(t) \cos st \cos sx ds dt$$

Q.5. Solve  $\frac{\partial v}{\partial t} = 2 \frac{\partial^2 v}{\partial x^2}$

If  $v(0,t) = 0, v(x,0) = e^{-x}, x > 0, v(x,t)$  is bounded where  $x > 0, t > 0$

OR

Use finite fourier transform to solve

$$\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial x^2}, \quad 0 < x < 6, t > 0 \text{ and}$$

$$V_x(0,t) = 0, V_x(6,t) = 0, V(x,0) = 2x$$

Q.6. If  $\bar{f}(s)$  and  $\bar{g}(s)$  are the Hankel transforms of the function  $f(x)$  and  $g(x)$  respectively, then prove that

$$\int_0^{\infty} x f(x) \cdot g(x) dx = \int_0^{\infty} s \bar{f}(s) \bar{g}(s) ds$$

(9)

OR

Define Mellin transform of the function  $f(x)$  and find  $M\{\sin x\}$ .

Section - C

Long Answer Type Questions

5 × 9 = 45

Q.7. Find the fourier transform of  $f(x)$  defined by

$f(x) = \begin{cases} 1, & |x| \leq a \\ 0, & |x| > a \end{cases}$  and hence evaluate

$$\int_{-\infty}^{\infty} \frac{\sin sa \cdot \cos s^x}{s} ds$$

OR

If  $F\{f(x)\}$  and  $F\{g(x)\}$  are the fourier transforms of functions  $f(x)$  and  $g(x)$  respectively, then prove that  $F\{f(x) * g(x)\} = F\{f(x)\} \cdot F\{g(x)\}$

(10)

Q.8. If  $f(x)$  satisfies the following conditions:

- (a)  $f(x)$  satisfies the Dirichlet conditions in every interval  $-l \leq x \leq l$ .
- (b)  $\int_{-\infty}^{\infty} |f(x)| dx$  converges, i.e  $f(x)$  is absolutely integrable in the interval  $-\infty < x < \infty$ , then prove that

$$f(x) = \frac{1}{2\pi} \int_{s=-\infty}^{\infty} \int_{t=-\infty}^{\infty} f(t) \cos s(x-t) ds dt$$

OR

If the fourier series corresponding to  $f(x)$  converges inifrmly to  $f(x)$  in the interval  $-l < x < l$ , then prove that

$\frac{1}{l} \int_{-l}^l [f(x)]^2 dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty} (a_n^2 + b_n^2)$  where the integral an L.H.S. is supposed to exists.

Q.9. By using parseval's identity, prove that

$$\int_0^{\infty} \frac{dt}{(a^2 + t^2)(b^2 + t^2)} = \frac{\pi}{2ab(a+b)}$$

(11)

OR

Express the function

$$f(x) = \begin{cases} 1, & \text{for } |x| \leq 1 \\ 0, & \text{for } |x| > 1 \end{cases} \text{ as a fourier integral and}$$

hence evaluate  $\int_0^{\infty} \frac{\sin \lambda \cos \lambda x}{\lambda} d\lambda$ .

Q.10. Use the method of fourier transform to determine the displacement  $y(x,t)$  of an infinite string, given that the string is initially at rest and that the initial displacement is  $f(x), -\infty < x < \infty$ , and show that the solutions can also be put in the form

$$y(x,t) = \frac{1}{2} [f(x+t) + f(x-t)].$$

OR

Using the fourier sine transform, solve the

equation  $\frac{\partial v}{\partial t} = k \frac{\partial^2 v}{\partial x^2}$  for  $x > 0, t > 0$ , under the

boundary conditions  $v = v_0$  when  $x = 0, t > 0$  and the initial condition  $v = 0$  when  $t = 0, x > 0$ .

(12)

Q.11. Find the Hankel transform of

$$\frac{d^2 f}{dx^2} + \frac{1}{x} \frac{df}{dx} - \frac{n^2}{x^2} f.$$

OR

Find fourier sine and cosine series of the function

$$f(x) \begin{cases} x, & 0 \leq x \leq \pi/2 \\ \pi - x, & \pi/2 < x \leq \pi \end{cases}$$



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