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### PK-478

## M.Sc. IV Semester Mathematics (Reg./Pvt./ATKT) Examination June 2018

### INTEGRAL TRANSFORM - II

(Optional Paper Select any four)

#### Paper - X

Time Allowed : Three Hours] [Maximum Marks : { Reg. - 85

Note : Attempt all questions.

{ Pvt. - 100

#### Section - A

#### Objective Type Questions

Q.1. Choose the correct answer: 15x1=15

i) The sine transform of  $f(x) = \frac{1}{x}$  is

(a)  $\sqrt{\pi}$  (b)  $\sqrt{\frac{2}{\pi}}$

(c)  $\frac{\pi}{2}$  (d)  $\sqrt{2\pi}$

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(2)

ii) The Fourier Transform of

$$f(x) = \begin{cases} 1-x^2, & |x| \leq 1 \\ 0 & |x| > 1 \end{cases} \text{ is.}$$

- (a)  $e^{-x}$
- (b)  $e^{-x}$
- (c) 1
- (d) None of these

iii) The cosine transform of  $e^{-x}$  is

- (a)  $\frac{s}{1+s^2}$  (b)  $\frac{1}{1+s^2}$
- (c)  $\frac{1}{1-s^2}$  (d)  $\frac{s}{1-s^2}$

iv) The finite Fourier cosine transform of  $x, 0 < x < \pi$  is :

- (a)  $\frac{\pi}{n}$  (b)  $-\frac{\pi}{n}(-1)^n$
- (c)  $-\frac{\pi}{n}$  (d) None of these

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Contd...

(3)

v) If  $f(s)$  is the Fourier transform of  $F(x)$

then  $\frac{1}{a} f\left(\frac{s}{a}\right)$  is the Fourier Transform of

(a)  $F\left(\frac{x}{a}\right)$

(b)  $F\left(\frac{a}{x}\right)$

(c)  $F(ax)$

(d) None of these

vi) The expression  $F_s\{f(x)\} = \int_0^l \sin \frac{p\pi x}{l} dx$  is called

- (a) Finite Fourier sine transform
- (b) Infinite Fourier sine transform
- (c) Finite Fourier cosine transform
- (d) Fourier complex transform

(4)

vii) The cosine transform of  $\frac{1}{1+s^2}$  is

(a)  $\frac{\pi}{2}$

(b)  $e^{-s}$

(c)  $\frac{\pi}{2} e^{-s}$

(d)  $e^{-s}$

viii) If  $F\{F(x)\} = f(s)$  then  $F\left\{\frac{d^n F}{dx^n}\right\}$  is

(a)  $s^n f(s)$

(b)  $(is)^n f(s)$

(c)  $s^n f^n(s)$

(d)  $(i \cdot s)^n f^n(s)$

ix) In Parseval identity for Fourier series,  $f(x)$  is

- (a) convergent
- (b) divergent
- (c) uniform convergent
- (d) oscillatory

(5)

x) If  $f(s)$  is the Fourier transform of  $F(x)$ , then  $e^{-asi} f(s)$  is the Fourier transform of

- (a)  $F(ax)$                       (b)  $F(x-a)$
- (c)  $F(a-x)$                     (d) None of these

xi) In case of infinite Fourier Transform exclusion of  $\frac{\partial^2 u}{\partial x^2}$  from a differential equations

- (a)  $\left(\frac{\partial u}{\partial x}\right)_{x=0}$  in cosine form,  $(u)_{x=0}$  in sine form
- (b)  $\left(\frac{\partial u}{\partial x}\right)_{x=0}$  in cosine form  $(u)_{x=0}$  in cosine form
- (c)  $\left(\frac{\partial u}{\partial x}\right)_{x=0}$  in sine form  $(u)_{x=0}$  in sine form
- (d)  $\left(\frac{\partial u}{\partial x}\right)_{x=0}$  in sine form  $(u)_{x=0}$  in cosine form

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(6)

xii)  $H\left(\frac{e^{-5x}}{x}, n=0\right) =$

- (a)  $\sqrt{25+s^2}$
- (b) 1
- (c)  $\frac{1}{\sqrt{25+s^2}}$
- (d) None of the above

xiii)  $M(e^{-x}) =$

- (a)  $\sqrt{p+1}, p > 0$       (b) 1
- (c)  $\sqrt{p}, p < 0$         (d)  $\sqrt{p}, p > 0$

xiv)  $M\left[\frac{1}{x} f\left(\frac{1}{x}\right)\right] =$

- (a)  $\overline{f(1-p)}$                       (b)  $\overline{f(1+p)}$
- (c)  $\frac{\overline{f(1-p)}}{(1+p)}$                       (d)  $\frac{\overline{f(1-p)}}{(1-p)}$

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(7)

xy) If  $f(2\pi - x) = f(x)$ , then Fourier series is

- (a) Fourier cosine series
- (b) Fourier sine series
- (c) Half range series
- (d) None of these

**Section - B**

**Short Answer Type Questions**

5 × 5 = 25

Q.2. Write relation between Fourier transform and Laplace transform.

**OR**

Find the Fourier transform of

$$f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$$

Q.3. State and prove Modulation theorem for Fourier transform.

**OR**

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(8)

Use Parseval's identities, prove

$$\int_0^{\infty} \frac{dt}{(a^2 + t^2)(b^2 + t^2)} = \frac{\pi}{2ab(a+b)}$$

Q.4. Find finite Fourier sine and cosine transform of  $f(x) = x^2, 0 < x < 4$

**OR**

Show that the finite sine transform of  $\frac{x}{\pi}$  is

$$(-1)^{s+1} \frac{1}{s}$$

Q.5. Use finite Fourier transform to solve  $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$  given that  $u(0,t) = 0, u(\pi,t) = 0, u(x,0) = 2x$  where  $0 < x < \pi, t > 0$ .

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(9)

Use finite cosine transform to solve

$$\frac{\partial v}{\partial t} = k \frac{\partial^2 v}{\partial x^2}, 0 < x < \pi, t > 0 \text{ with the boundary}$$

conditions  $\frac{\partial v}{\partial x} = 0$  when  $x = 0$  and  $x = \pi, t > 0$  and

the initial condition  $V = f(x)$  when  $t = 0, 0 < x < \pi$ .

Q.6. Find the Hankel transform of  $x^{-2}e^{-x}$  of order one.

OR

$$\text{Show that } M(\sin x) = \sqrt{p} \sin \frac{p\pi}{2}.$$

**Section - C**  
**Long Answer Type Questions**

5×9=45

Q.7. Find the Fourier transform of

$$F(x) = \begin{cases} 1-x^2, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$$

and hence evaluate

$$\int_0^{\infty} \frac{(x \cos x - \sin x)}{x^3} \cos \frac{x}{2} dx$$

OR

(10)

Find cosine transform of  $\frac{1}{1+x^2}$

Q.8. Find the finite cosine transform of  $\left(1 - \frac{x}{\pi}\right)^2$

OR

Find the finite Fourier sine transform of  $f(x)$  where  $f(x) = \cos kx$ .

Q.9. State and prove Rayleigh Theorem.

OR

Find  $f(x)$  if its Fourier cosine transform is

$$\frac{1}{1+s^2}.$$

Q.10. Using the Fourier sine transform, solve the partial

differential equation  $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$ .

Under the boundary conditions  $u = u_0$  when  $x = 0, t > 0$ , and initial condition  $u = 0$  when  $t = 0, x > 0$ .

OR

(11)

Use the method of Fourier transform to determine the displacement  $y(x, t)$  of an infinite string, given that the string is initially at rest and the initial displacement is  $f(x)$ ,  $-\infty < x < \infty$  show that the solution can also be put in the form

$$y(x, t) = \frac{1}{2} [f(x + ct) + f(x - ct)].$$

Q.11. Obtain the Fourier series for  $e^x$  in the interval  $[-\pi, \pi]$ . What is the sum of the series for  $x = \pm \pi$ . <http://www.onlinebu.com>

OR

Find Fourier sine and cosine series of the function

$$f(x) = \begin{cases} x & , 0 \leq x \leq \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} < x \leq \pi \end{cases}$$



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