AS-466

M.Sc. (Maths) II Semester
(Reg./Pvt./ATKT) Examination July 2019

TOPOLOGY - II

Paper - III

Time Allowed : Three Hours] *[Maximum Marks :* { *Reg.-85* / *Pvt.-100*

Note : Attempt all questions.

Section - A
Objective Type Questions

$15 \times 1 = 15$

Q.1. Choose correct answer:

i) Which is not true in the following.

(a) A compact sub space of a Hausdorff space is always closed.

(b) A closed sub space of the normal space is normal.

(c) Every T_1-space is a T_2-space.

(d) None of these

ii) A class of sub sets of a non-empty set is said to have the finite intersection property if.

(a) Every finite sub class has non-empty intersection.

(b) A finite sub class has non-empty intersection

(c) Every finite sub class has empty intersection.

(d) None of these

iii) Which is not true in the following:

(a) R is disconnected

(b) R is connected

(c) Each point is a component

(d) None of these

iv) A subset A of a space X is closed iff

(a) Limits of nets in A are in A

(b) Limits of nets in A are in A'

(c) Limits of nets in A' are in A

(d) None of these

v) If f is a path in X from x_0 to x_1 and g is a path in X from x_1 to x_2 then

(a) $[f]*[g]$ is defined for every pair of path homotopic classes.

(b) $[f]*[g]$ is defined for those pair of path homotopic classes for which $f(0)=f(1)$.

(c) $[f]*[g]$ is defined for those pair of path homotopic classes $f(1)=f(0)$

(d) None of these

vi) Which of the following is not true.

(a) Any infinite sub set A in a discrete topological space is compact

(b) Every indiscrete space is compact

(c) Every topological space (x,T) is compact if X is finite

(d) None of these

vii) Every topological space (X,T) is compact if

(a) X is not finite

(b) T is not finite

(c) Every basic open cover of X has a finite sub cover

(d) None of these

viii) Which one of the following is correct

(a) Every discrete space is a Hausdorff space

(b) Every T_0-space is a Hausdorff space

(c) Every T_1- space is a Hausdorff space

(d) None of these

ix) Let (X,T) be a T_3-spce, then which one is true?

(a) (X,T) is a T_2-space

(b) (X,T) is a T_1-space

(c) (X,T) is a T_4-space

(d) None of these

x) Two sets A and B are not separated sets if

(a) A = (2,3) and B = (3,4)

(b) A = (2,3) and B = (4,5)

(c) A = (3,4) and B = (4,5)

(d) None of these

xi) A topological space (X,T) is disconnected if there exists two open subsets G and H of X such that.

(a) $G \cap H \neq \phi$

(b) $G \cup H \neq X$

(c) $G = \phi$ but $H \neq \phi$

(d) None of these

xii) Let (X,T) be a topological space and let $S:D \to X$ be a net. Then S is said to converge to a point $x \in X$ if given any open set $\cup$ containing x, there exists $m \in D$ such that

(a) for all $n \in D,\ n \geq m \Rightarrow Sn \in \cup$

(b) for all $n \in D,\ n \geq m \Rightarrow Sm \in \cup$

(c) for all $n \in D, m \geq n \Rightarrow Sn \in \cup$

(d) None of these

xiii) Which is true in the following?

(a) Some filters has no finite intersection property

(b) Every filter has finite intersection property.

(c) A family having finite intersection property is a filter.

(d) None of these

xiv) Which is true in the following:

(a) The set of path-homotopy classes of paths in a space X form a group under the operation *.

(b) The set of path homotopy classes of loops based at x_0, with the operation *, form a group.

(c) The set of path homotopy classes of loops based at x_0, with the operation *, does not form a group.

(d) None of these

xv) A space X is said to be simply connected is

(a) it is a path connected space

(b) if $\Pi_1(X,x_0)$ is the trivial group for some $x_0 \in X$

(c) It is a path connected space and if $\Pi_1(X,x_0)$ is the trivial group for some $x_0 \in X$

(d) None of these

Section - B

Short Answer Type Questions

$5 \times 5 = 25$

Q.2. If $T = \{\phi, X, \{a\}, \{a,b\}, \{a,c\}\}$ is a topology on $X = \{a,b,c\}$. Then show that the topological space (X,T) is not a T_1-space.

OR

Define completely Regular space and Normal space.

Q.3. Define compact space and give an example of it.

OR

Define sequentially compact space and Bolzano-weierstrass property.

Q.4. Define projection mappings. If (X,T) is the product of topological spaces (X_1,T_1) and (X_2,T_2) then prove that the projection map π_1 is continuous.

OR

If the product space $X_1 \times X_2$ is connected then prove that X_1 and X_2 are connected.

Q.5. Define filter and also define limit of a filter.

OR

Let S be a family of subsets of a set X, then prove that there exists a filter on X having S as a subbase if and only if S has the finite intersection property.

Q.6 Define homotopic, homotopy. path homotopy.

OR

If $P: E \to B$ and $P': E' \to B'$ are covering maps, then prove that $P \times P' : E \times E' \to B \times B'$ is a covering map.

Section - C

Long Answer Type Questions

$5 \times 9 = 45$

Q.7. Prove that a one-to-one mapping of a compact space onto a Hausdorff space is a homeomorphism.

OR

Let X be a normal space, and let A and B be disjoint closed subspaces of X. Then prove that there exists a continuous real function f defined on X, all of whose values lie in the closed unit interval [0,1] such that $f(A) = 0$ and $f(B) = 1$.

Q.8. Define open cover. Prove that any closed subspace of a compact space is compact.

OR

Define E-net. Prove that every sequentially compact metric space is tatally bounded.

Q.9. Define Hausdorff space. Prove that the product of any non-empty class of Hausdorff spaces is a Hausdorff space.

OR

Prove that the spaces R^n and C^n are connected.

Q.10. Let X any y be topological spaces, $x_0 \in X$ and $f: X \to y$ a function. Then prove that f is continuous at x_0 if and only if whenever a net s converges to x_0 in X, then net fos converges to f (x_0) in y.

OR

Prove that a topological space is compact is and only if every ultrafilter in it is convergent.

Q.11. Define "α-hat". Prove that the map $\hat{\alpha}$ is a group Isomorphism.

OR

Prove that a polynomial equation $x^n + a_{n-1}x^{n-1} + \ldots\ldots + a_1x + a_0 = 0$ of degree n > 0 with real or complex coefficients has at lest one (real or complex) root.

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