

Total No. of Questions : 11
Roll No. :
Total No. of Printed Pages : 7

AS-465

**M.Sc. (Maths) II Semester
(Reg./Pvt./ATKT) Examination July 2019**

**LEBESQUE MEASURE AND
INTEGRATION-II**

Paper - II

Time Allowed : Three Hours] Maximum Marks : $\begin{cases} \text{Reg. - 85} \\ \text{Pvt. - 100} \end{cases}$

Note : Attempt all questions.

Section - A

Objective Type Questions

5×3=15

Q.1. i) The outer measure of (a, b) is given by

- (a) a + b
- (b) b - a
- (c) b² - a²
- (d) None of these

YA19-218

AS-465

P.T.O.

(2)
ii) The value of $\int_0^1 \sin x \log x \, dx$ is

- (a) $\sum n^2$
- (b) $\sum \frac{(-1)^n}{(2n)^2}$
- (c) $\sum \frac{(-1)^n}{(2n)(2n)!}$
- (d) None of these

iii) If $f(x) = |x|$ then the value of D^+ at $x = 0$ is :

- (a) 1
- (b) 0
- (c) -1
- (d) 2

YA19-218

AS-465

Contd...

(3)

iv) Which is not correct :

- (a) e^x is strictly convex on \mathbb{R}
- (b) x^α is convex on $(0, \infty)$ for $\alpha \geq 1$
- (c) $-x^\alpha$ is strictly convex on $(0, \infty)$ for $0 < \alpha < 1$
- (d) $x \log x$ is not convex on $(0, 1)$

v) Let $\{f_n\}$ be a sequence of measurable functions and f a measurable function. Then f_n tends to f in measure if for every positive ϵ :

- (a) $\lim [x : |f_n(x) - f(x)| > \epsilon] = 0$
- (b) $\lim [x : |f_n(x) - f(x)| > \epsilon] = \infty$
- (c) $\lim [x : |f_n(x) - f(x)| < \epsilon] = -\infty$
- (d) None of these

http://www.onlinebu.com

http://www.onlinebu.com

(4)

Section - B

Short Answer Type Questions

5x5=25

Q.2. Show that outer measure is translation invariant for any sequence of sets $\{E_i\}$, prove that

$$m^*\left(\bigcup_{i=1}^{\infty} E_i\right) \leq \sum_{i=1}^{\infty} m^*(E_i)$$

Q.3. Show that if f is a non-negative measurable function, then $f=0$ a.e. if and only if $\int f dx = 0$

OR

Show that if f and g are measurable, $|f| \leq |g|$ a.e., and g is integrable then f is integrable.

Q.4. Let f be defined by $f(x) = x \sin\left(\frac{1}{x}\right)$, for $x \neq 0$ and $f(0) = 0$, then find the four derivatives at $x = 0$.

OR

Define function of bounded variation.

http://www.onlinebu.com

http://www.onlinebu.com

(5)

Q.5. Let $f, g \in L^p(\mu)$ and let a, b be constants, then prove that $af + bg \in L^p(\mu)$.

OR

Define convex function.

Q.6. If $f_n \rightarrow f$ a.u., then prove that $f_n \rightarrow f$ in measure.

OR

If $f_n \rightarrow f$ a.u., then prove that $f_n \rightarrow f$ a.e.

Section - C

Long Answer Type Questions

5×9=45

Q.7. Prove that the class M is a σ - algebra.

OR

Let C be any real number and let f and g be real valued measurable functions defined on the same measurable set E . Then prove that $f + c, cf, f + g, f - g$ and fg are also measurable.

YA19-218

AS-465

P.T.O.

(6)

Q.8. State and prove Fatou's Lemma.

OR

State and prove Lebesgue's Monotone convergence theorem.

Q.9. State and prove Lebesgue's Differentiation theorem.

OR

If f is a finite - valued monotone increasing function defined on the finite interval $[a, b]$, then prove that f is measurable and

$$\int_a^b f' dx \leq f(b) - f(a)$$

Q.10. State and prove Jensen's inequality.

OR

State and prove Holder's inequality.

YA19-218

AS-465

Contd...

http://www.onlinebu.com

http://www.onlinebu.com

http://www.onlinebu.com

http://www.onlinebu.com

(7)

Q.11. Let $\{f_n\}$ be a sequence of non-negative measurable functions and let f be a measurable function such that $f_n \rightarrow f$ in measurable then prove that

$$\int f d\mu \leq \liminf \int f_n d\mu$$

OR

Let $f_n \rightarrow f$ a.e. If

i) $\mu(X) < \infty$

OR

ii) For each n , $|f_n| \leq g$, an integrable function, then prove that

$$f_n \rightarrow f \text{ a.u.}$$



http://www.onlinebu.com

Whatsapp @ 9300930012

Your old paper & get 10/-

पुराने पेपर्स भेजे और 10 रुपये पायें,

Paytm or Google Pay से

AS-465

YA19-218