

**AS-467**

**M.Sc. (Maths) II Semester  
(Reg./Pvt./ATKT) Examination July 2019**

**COMPLEX ANALYSIS - II**

**Paper - IV**

Time Allowed : Three Hours] [Maximum Marks : { Reg.-85  
Pvt.-100

Note : All the questions are compulsory.

**Section - A  
Objective Type Questions**

15 × 1 = 15

Q.1. Choose the correct answer:

- i) The power series  $f(z) = \sum_{n=0}^{\infty} z^n$
- (a) Converges when  $|z| < 1$
  - (b) Converges when  $|z| > 1$
  - (c) Diverges when  $|z| < 1$
  - (d) None of these

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ii) Infinite product  $\prod_{n=1}^{\infty} \left(1 - \frac{z^2}{n^2}\right)$  represents

- (a)  $\sin z$
- (b)  $\sin \pi z$
- (c)  $\cos z$
- (d)  $\cos \pi z$

iii)  $G(-z) =$

- (a)  $\prod_{n=1}^{\infty} \left(1 - \frac{z}{n}\right)$
- (b)  $\prod_{n=1}^{\infty} \left(1 + \frac{z}{n}\right) e^z$
- (c)  $\prod_{n=1}^{\infty} \left(1 - \frac{z}{n}\right) e^{\frac{z}{n}}$
- (d)  $\prod_{n=1}^{\infty} \left(1 - \frac{z}{n}\right) e^{-\frac{z}{n}}$

iv) An analytic function f with its domain of definition D is called:

- (a) Germ
- (b) Harmonic
- (c) Function element
- (d) None of these

v) Analytic continuation of  $f_1(z) = \int_0^{\infty} e^{-zt} dt =$

(a)  $f_2(z) = \sum_{n=0}^{\infty} z^n$

(b)  $f_2(z) = \sum_{n=0}^{\infty} \frac{(z+i)^n}{i}$

(c)  $f_2(z) = \sum_{n=0}^{\infty} \frac{(z+i)^n}{i^{n-1}}$

(d) None of these

vi) The number of analytic continuation of a function  $f(z)$  into the same domains is:

(a) 1

(b) 2

(c) 3

(d) None of these

vii) For Poisson kernel  $P_r(\theta)$

(a)  $\frac{1}{2\pi} \int_{-\infty}^{\infty} P_r(\theta) d\theta = 0$

(b)  $\frac{1}{2\pi} \int_{-\infty}^{\infty} P_r(\theta) d\theta < 1$

(c)  $\frac{1}{2\pi} \int_{-\infty}^{\infty} P_r(\theta) d\theta \leq 1$

(d)  $\frac{1}{2\pi} \int_{-\infty}^{\infty} P_r(\theta) d\theta \geq 1$

viii) For Green's function  $g_a$  which of the following is not true:

(a)  $g_a$  is harmonic in  $G - \{a\}$

(b)  $G(z) = g_2(z) + \log|z-a|$  is harmonic in disc about  $a$

(c)  $\lim_{z \rightarrow \infty} g_a(z) = 0$  for each  $\infty$  in  $\partial_{\infty}G$

(d) None of these

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ix) A continuous function  $u: G \rightarrow R$  has the mean Value Property (MVP) if whenever  $\bar{B}(a, r) \subset G$ ,  $u(a) =$

(a)  $\int_0^{2\pi} u(a + re^{T\theta}) d\theta$

(b)  $\frac{1}{2\pi} \int_0^{2\pi} u(a + re^{i\theta}) d\theta$

(c)  $\int_0^{2\pi} re^{T\theta} d\theta$

(d) None of these

x) The order of the function  $\cos z$  is

(a)  $\infty$

(b) 0

(c) 1

(d) -1

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xi) The canonical product of  $\cos hz =$

(a)  $\prod_{n=1}^{\infty} \frac{z^2}{(2n-1)^2 z^2}$

(b)  $\prod_{n=1}^{\infty} \left[ 1 + \frac{4z^2}{(2n-1)^2 \pi^2} \right]$

(c)  $\prod_{n=1}^{\infty} \left( 1 + \frac{z^2}{n^2 z^2} \right)$

(d) None of these

xii) The convergence of exponent  $\sigma$  of a sequence  $\{z_n\}$  is given by:

(a)  $\sigma = \limsup_{n \rightarrow \infty} \frac{\log n}{\log |z_n|}$

(b)  $\sigma = \liminf_{n \rightarrow \infty} \frac{\log n}{\log |z_n|}$

(c)  $\sigma = \limsup_{n \rightarrow \infty} \log |z_n|$

(d) None of these

xiii) "Let  $f$  be an entire function that units two values, then  $f$  is a constant." This statement is:

- (a) The great Picard theorem
- (b) The little Picard theorem
- (c) Schollky's theorem
- (d) Borel's theorem

xiv) If  $f(z)$  is a univalent function in a domain  $D$  then the inverse mapping  $z = g(\infty)$  is :

- (a) Univalent
- (b) Not univalent
- (c) Both
- (d) None of these

xv) The number  $B = \inf \{ \beta(f) : f \in F \}$  is called

- (a) Landau's constant
- (b) Bord's constant
- (c) Bloch's constant
- (d) None of these

Section - B

Short Answer Type Questions

5 × 5 = 25

Q.2: State Weierstrass factorization theorem.

OR

Prove that  $\prod_z \overline{1-z} = \frac{\pi}{\sin \pi z}$

Q.3. Define the following:

- a) Analytic continuation
- b) A function element

OR

Find the analytic continuation of the function  $f_1$

defined by the series  $f_1(z) = \sum_{n=0}^{\infty} z^n$

Q.4. Define Harmonic functions and write their basic properties.

OR

If  $u : G \rightarrow R$  is a continuous function which has the mean value property then show that  $u$  is harmonic.

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Q.5. Find the order of the polynomial

$$P(z) = a_0 + a_1z + a_2z^2 + \dots + a_n z^n, a_n \neq 0.$$

OR

Let  $f(z)$  be an entire function with  $f(0) \neq 0$  Also let  $r_1, r_2, \dots, r_n$  be the modulo of zeros  $z_1, z_2, \dots, z_n$  of  $f(z)$ , arranged as non decreasing sequence, multiple zero being repeated. then prove that

$$R^n |f(0)| \leq m(R)r_1, r_2, \dots, r_n$$

Where  $r_n < R < r_{n+1}$ , and  $m(R)$  is the max. modulus of  $f(z)$  on  $|z| = R$

Q.6. Explain :

- Univalent function
- Bloch's constant

OR

Let  $f$  be analytic in  $D = \{z : |z| < 1\}$  and let  $f(0) = 0, f'(0) = 1, |f(z)| \leq m \forall z$  in  $D$ .

Then prove that  $m \geq 1$  and  $f(D) \supset B\left(0, \frac{1}{6m}\right)$

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Section - C

Long Answer Type Questions

5 × 9 = 45

Q.7. For  $R, z > 1$  prove that

$$\xi(z) \Gamma(z) = \int_0^\infty (e^t - 1)^{-1} t^{z-1} dt$$

OR

$$\text{Prove that } \sqrt{\pi} \Gamma(2z) = 2^{2z-1} \Gamma(z) \Gamma\left(z + \frac{1}{2}\right)$$

Q.8. State and prove Mittag Leffter theorem.

OR

Let  $y : [0, 1] \rightarrow \mathbb{C}$  be a path and let

$\{(f_t, D_t) : 0 \leq t \leq 1\}$  be an analytic continuation along  $y$ . for  $0 \leq t \leq 1$ , let  $R(t)$  be the radius of convergence of the power series expansion of  $f_t$  about  $z = y(t)$ . Then prove that either  $R(t) \equiv \infty$  or  $R : [0, 1] \rightarrow (0, \infty)$  is continuous.

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Q.9. Let  $f(z)$  be a function of  $z$  analytic in a domain  $D$  which contains a segment of  $x$ -axis about which  $D$  is symmetric. Then prove that

$\overline{f(z)} = f(\bar{z}), z \in D$  i.e.  $f(z)$  takes conjugate values for conjugate values of  $z$  if and only if  $f(x)$  is real for each point on the segment of  $x$ -axis.

OR

State and prove Harnack's inequality.

Q.10. State and prove Borel's theorem

OR

State and prove Hadamard's factorization theorem

Q.11. State and prove Montel-Caratheodory theorem.

OR

State and prove the  $\frac{1}{4}$  theorem.

