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Total No. of Questions : 11} [Total No. of Printed Pages : 7

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**M.Sc. (Maths.) II Semester  
(Reg./Pvt./ATKT) Examination July 2019  
ADV. ABSTRACT ALGEBRA - II**

Paper - I

Time Allowed : Three Hours} [Maximum Marks : { Reg.-85  
Pvt.-100

Note : All questions are compulsory.

**Section - A**

**Objective Type Questions**

Q.1. Choose the correct answer. 5×2=10

- i) If  $M$  be an  $R$ -module, a non-empty subset  $N$  of  $M$  is called an  $R$ -submodule of  $M$  if :
  - (a)  $a - b \in N \forall a, b \in N$
  - (b)  $r \in R, a \in N \Rightarrow ra \in N$
  - (c) Both (a) and (b)
  - (d) None of these

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(2)

ii) An  $R$ -sub module  $N$  of  $M$  is maximal in  $M$  iff:

- (a)  $M$  is simple
- (b)  $N$  is simple
- (c)  $M + N$  is simple
- (d)  $M/N$  is simple

iii) Every homomorphic Image of an Artinian module is :

- (a) Artinian
- (b) Noetherian
- (c) Both (a) & (b)
- (d) None of these

iv) A non-zero module  $M$  is called uniform if any two sub modulus of  $M$  have :

- (a) Zero intersection
- (b) Non-zero intersection
- (c) Empty intersection
- (d) All of these

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(3)

v) The element  $\lambda \in F$  is a characteristic root of  $T \in A(V)$  iff for some :

- (a)  $V \neq 0, VT = \lambda V$
- (b)  $V \neq 0, V^T/\lambda V$
- (c)  $V \neq 0, \lambda V/VT$
- (d) None of these

**Section - B**

**Short Answer Type Questions**

5x5=25

Q.2. Define Quotient module and prove that the sub module of the Quotient module.

OR

Let  $f: M \rightarrow N$  be a R-homomorphism of an R-Module M into an R-module N. Then prove that  $\ker f$  is a R-sub-module of M.

Q.3. Define free module with two example.

OR

State and prove Schur's Lemma.

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Q.4. Prove that every homomorphic Image of a Noetherian module is Noetherian.

OR

Prove that a R-module M is noetherian iff every submodule of M is finitely generated.

Q.5. Write the statement of Noether Lasker theorem.

OR

Define uniform module with example.

Q.6. If T be a linear operator on  $R^2$ , the matrix, of which standard ordered basis of  $B = \{e_1, e_2\}$ , when  $e_1 =$

$$(1, 0), e_2 = (0, 1) \text{ is } A = \begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix}$$

OR

Define matrix of Linear Transformation.

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Contd...

(5)

(6)

Section - C

Long Answer Type Questions

5×10=50

Q.7. Define kernel of homomorphism and prove that if  $N$  be a sub module of an  $R$ -module  $M$ . Let  $f: M \rightarrow M/N$  be a mapping from the  $R$ -module  $M$  into the quotient module  $M/N$  defined by  $f(x) = x + N, \forall x \in M$ . Then prove that  $f$  is an  $R$ -homomorphism of  $M/N$  onto and  $\ker f = N$ .

OR

Let  $R$  be a Ring with unity, then show that an  $R$ -module  $M$  is cyclic iff  $M \cong R/I$  for some Left Ideal  $I$  of  $R$ .

Q.8. Let  $M$  be a finitely generated free module over a commutative Ring  $R$ , then prove that all bases of  $M$  have the same number of elements.

OR

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Let  $R$  be a ring with unity and let  $M$  be an  $R$ -module, then prove that following statement are equivalent.

- i)  $M$  is simple
- ii)  $M \neq 0$  and  $M$  is generated by any  $0 \neq x \in M$
- iii)  $M \cong R/I$ , where  $I$  is maximal left ideal of  $R$ .

Q.9. State and prove Hilbert basis theorem.

OR

State and prove Wedderburn Artin-theorem.

Q.10. If  $M$  be a non zero finitely generated module over a commutative nonetherian ring  $R$ , then prove that there are only a finite member of primes associated with  $M$ . <http://www.onlinebu.com>

OR

Show that in a left (right) Artinian Ring, Every nil (left (right) Ideal is Nilpotent.

Q.11. Let  $V$  be an  $n$ -dimensional vector space over field  $F$  and let  $T \in A(V)$  has all its characteristic roots (eigen value) in  $F$ . Then there is a basis of  $V$  in which the matrix of  $T$  is triangular.

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(7)

OR

For any non-zero  $T \in A(V)$ , prove that there exists a unique monic polynomial  $m(x) \in F[x]$ , such that

- i)  $m(T) = 0$
- ii) For any  $g(x) \in F(x)$ ,  $g(T) = 0$ , if and only if  $m(x) \mid g(x)$
- iii)  $F(T) = \frac{F[x]}{(m(x))}$



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