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Total No. of Questions : 11] [Total No. of Printed Pages : 4

PK-454

M.Sc. II Semester Mathematics
(Reg./Pvt./ATKT) Examination June 2018
TOPOLOGY - II

Paper - III

Time Allowed : Three Hours] *[Maximum Marks : { Reg. - 85, Pvt. - 100*

Note : Attempt all questions.
All questions are compulsory.

Section - A
Objective Type Questions

Q.1. Choose the correct answer: 5×2=10

i) A topological space X is a Hausdorff space is for every pair of distinct points x, y in X.
   (a) there exists open sets U and V in X such that $x \in U$ and $y \in V$
   (b) there exists disjoint open sets U and V in X such that $x \in U$ and $y \in V$
   (c) there exists open sets U and V in X such that $x, y \in U \cap V$
   (d) None of these

ii) Every sequentially compact metric space
   (a) is totally bounded
   (b) has an $\in$-net for each $\in > 0$
   (c) is compact
   (d) all the above

iii) A topological space X is connected if it can not be represented as a union of
   (a) two disjoint non-empty open sets
   (b) two disjoint non-empty closed sets
   (c) two non-empty open sets
   (d) both (a) and (b)

iv) A filter F on a set X is an ultrafilter if
   (a) it is properly contained in any filter on X
   (b) it is not properly contained in any filter on X
   (c) it contains any filter on X
   (d) None of these

v) The set of path homotopy classes in a space X.
   (a) forms a group under the operation $*$
   (b) forms a semigroup under the operation $*$
   (c) forms a monoid under the operation X
   (d) None of these

Section-B
Short Answer Type Questions

5×5=25

Q.2. Prove that a topological space is a T_1-space if and only if each point is a closed set.

OR

What do you mean by C(X, R) separates points. If C(X, R) separates points, then show that X is a Hausdorff space.

Q.3. Prove that a topological space is compact if every basic open cover has a finite sub cover.

OR

Define locally compact space. Prove that any compact space is a locally compact space.

Q.4. Discuss product topology.

OR

Define connected space. Prove that the discrete two point space is disconnected.

Q.5. Suppose $S : D \to X$ is a net and F is a cofinal subset of S.gf $S/F : F \to X$ converges to a point x in X, then prove that x is a cluster point of S.

OR

Define convergence and limit of a net.

Q.6. Prove that the map $\hat{\alpha}$ is a group isomorphism.

OR

If f and g are any two continuous maps of a space X into R^2, then prove that the map $F(x, t) = (1-t) f(x) + tg(x)$ is a homotopy between them.

Section-C
Long Answer Type Questions

5×10=50

Q.7. Prove that in a Hausdorff space, any point and disjoint compact sub space can be separated by open sets, in the sense that they have disjoint neighborhoods.

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OR

Prove that every compact Hausdorff space is Normal.

Q.8. Prove that every closed and bounded sub space of the real line is compact.

OR

Prove that a metric space is sequentially compact if and only if it has the Bolzano-Weierstrass property.

Q.9. Prove that the product of any non-empty class of compact spaces is a compact space.

OR

Prove that the product of any non-empty class of connected spaces is a connected space.

Q.10. Let A be a subset of a space X and let $x \in X$. Then prove that $x \in \overline{A}$ if and only if there exists a net in A which converges to x in X.

OR

Prove that a topological space is a Hausdorff space iff no filter can converge to more than one point in it.

Q.11. Prove that the relations $\simeq$ and $\simeq p$ are equivalence relations.

OR

Prove that the fundamental group of S^1 is isomorphic to the additive group of integers.