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PK-455

**M.Sc. II Semester Mathematics
(Reg/Pvt./ATKT) Examination June 2018
COMPLEX ANALYSIS - II
Paper - IV**

Time Allowed : Three Hours]

[Maximum Marks : {
Reg. - 85
Pvt. - 100}

Note : All questions are compulsory.

**Section - A
Objective Type Questions**

- Q.1. Choose the correct answer. $5 \times 2 = 10$
- i) For zeta function $\xi(x)$, the points $z = -2, -4$ are called
 (a) Zeros of ξ
 (b) Fixed point of ξ
 (c) Trivial zeros of ξ
 (d) None of these

(2)

- ii) If $f(z)$ is an entire transcendental function with maximum modulus $m(r)$ then $\lim_{r \rightarrow \infty} \frac{\log m(r)}{\log r}$ is:
 (a) $\log r$
 (b) $\log m(r)$
 (c) $-\infty$
 (d) ∞
- iii) The order of function $e^{ax}, a \neq 0$ is
 (a) $\frac{1}{a}$ (b) a
 (c) ∞ (d) 1
- iv) An entire function f is of finite order if there is a positive constant a and an $r_0 > 0$ such that
 (a) $|f(z)| < \exp(|z|^a)$ for $|z| > r_0$
 (b) $|f(z)| > \exp(|z|^a)$ for $|z| > r_0$
 (c) $|f(z)| < \exp(|z|^a)$ for $|z| < r_0$
 (d) None of these

(3)

- v) If f is an analytic function on a region containing $\bar{B}(0; R)$ then $f(B(0; R))$ contains a disk of radius.

- (a) $\frac{1}{72}R|f(0)|$ (b) $\frac{1}{72}R|f'(0)|$
 (c) $\frac{1}{72}Rf'(0)$ (d) None of these

Section - B
Short Answer Type Questions

5×5=25

- Q.2. If $|z| \leq 1$ and $p \geq 0$ then prove that

$$|1 - E_p(z)| \leq |z|^{p+1}$$

where $E_p(z)$ is elementary factor.

OR

Explain the following:

- i) Riemann zeta function
 ii) Riemann's functional equation

(4)

- Q.3. Prove that there cannot be more than one analytic continuation of a function $f(z)$ into the same domain.

OR

Show that the series $\sum_{n=0}^{\infty} \frac{z^n}{2^{n+1}}$ and $\sum_{n=0}^{\infty} \frac{(z-i)^n}{(2-i)^{n+1}}$ are analytic continuation of each other.

- Q.4. Define Harmonic functions and write their basic properties.

OR

Define the following:

- i) Dirichlet's region
 ii) Green's function

- Q.5. Find the order of the function $f(z) = \cos z$.

OR

If $f(z)$ is an entire function of order λ and convergence exponent σ , then prove that $\sigma \leq \lambda$.

- Q.6. Define the following:

- i) Range of an analytic function
 ii) Univalent function

(5)

OR

Let f be an analytic function in a region containing the closure of the disc $D = \{z : |z| < 1\}$ and $f(0) = 0, f'(0) = 1$. Then prove that $f(D)$ contains a disc of radius?

Section - C

Long Answer Type Questions

$5 \times 10 = 50$

Q.7. Prove that

$$\sqrt{\pi} \Gamma(2z) = 2^{2z-1} \Gamma(z) \sqrt{z + \frac{1}{2}}$$

OR

State and prove Weierstrass factorization theorem.

Q.8. Find the analytic continuation of the function f_1 defined by the series

$$f_1(z) = \sum_{n=0}^{\infty} z^n$$

OR

(6)

State and prove Mittag Leffler's theorem.

Q.9. State and prove Schwarz's reflection principle.

OR

State and prove Harnack's theorem for Harmonic functions.

Q.10. State and prove Borel's theorem.

OR

State and prove Hadamard's three circles theorem.

Q.11. State and prove Bloch's theorem.

OR

State and prove the $\frac{1}{4}$ theorem.

