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PK-452

M.Sc. II Semester Mathematics (Reg./Pvt./ATKT) Examination June 2018

ADVANCED ABSTRACT ALGEBRA - II

Paper - I

Time Allowed : Three Hours] *[Maximum Marks : Reg. - 85, Pvt. - 100*

Note : All questions are compulsory.

Section - A
Objective Type Questions

Q.1. Choose the correct answer: 5×2=10

i) If M and N be two given R-modules then mapping $f: M \to N$ is said to be R-homomorphism if

(a) $f(m_1+m_2)=f(m_1)+f(m_2)$

(b) $f(m_1 m_2)=f(m_1)f(m_2)$

(c) $f\left(\frac{m_1}{m_2}\right)=\frac{f(m_1)}{f(m_2)}$

(d) None of these

ii) A finite sequence $y_1, y_2, \dots, y_n$ of elements of an R-module M is called Linearly Independent if :

(a) $\sum_{i=0}^{n} a_i y_i = 0,\ a_1 = a_2 = \dots = a_n = 0$

(b) $\sum_{i=0}^{n} a_i y_i = 0,\ a_1 \neq a_2 \neq \dots \neq a_n \neq 0$

(c) $\sum_{i=0}^{n} a_i y_i = 0,\ a_1 = a_2 \neq 0, a_3 = \dots = a_n = 0$

(d) None of these

iii) A non-zero module M is called uniform if any two non-zero sub modules of M have

(a) Non-zero Addition

(b) Non zero union

(c) Non-zero intersection

(d) None of these

iv) An R-module M is said to be finitely co-generated if:

(a) $\bigcap_{\alpha \in \wedge} M_\alpha = (0)$

(b) $\bigcap_{\alpha \in \wedge} M_\alpha \neq (0)$

(c) $\bigcup_{\alpha \in \wedge} M_\alpha = (0)$

(d) None of these

v) The element $\lambda \in F$ is a characteristic root of $T \in A(\upsilon)$ if and only if for some $\upsilon \neq 0$ in v :

(a) $\nu T \neq \lambda \nu$ (b) $\nu T = \lambda \nu$

(c) $\nu \lambda = \lambda T$ (d) None of these

Section - B

Short Answer Type Questions

5×5=25

Q.2. Define module and prove that Let $f : M \rightarrow N$ be an R-homomorphism of an R-module M into an R-module M into an R-module N. Then the Ker f is an R-submodule of M.

OR

Define sub module and prove that intersection of two sub modules is also sub module.

Q.3. Define simple module and semi simple module with example.

OR

State and prove that Schur's Lemma.

Q.4. Show that every submodule and every Quotient module of Noetherian module is Noetherian.

OR

Define Noetherian and Artinian rings with examples.

Q.5. Define uniform and primary module with example.

OR

If M be a Noetherian module over Noetherian Ring then prove that each non-zero sub module contains a uniform module.

Q.6. Find the Jordan canonical form of the matrix A.

$$A = \begin{bmatrix} 0 & 4 & 2 \\ -3 & 8 & 3 \\ 4 & -8 & -2 \end{bmatrix}$$

OR

For any $T_1, T_2 \in \text{Hom}(U, V)$,

Show that

$T_1 + T_2 \in \text{Hom}(U, V)$.

Section - C

Long Answer Type Questions

5×10=50

Q.7. State and prove fundamental theorem of R-homomorphism.

OR

Let R be a Euclidean ring, then any finitely generated R-module M is the direct sum of a finite number of cyclic sub-module.

Q.8. The following conditions on an R-module M are equivalent

a) M is a sum of simple sub-modules

b) M is semi-simple module

c) Every submodule of M is a direct summand of M.

OR

Define free module with example. If M be a free R-module with a basis $\{e_1, e_2, \ldots, e_n\}$ then show that $M = R^4$.

Q.9. Show that for an R-module M, the following are equivalent

a) M is noetherian

b) finiteness condition hold for M

c) Maximum condition hold for M

OR

Prove that every homomorphic image of an Artinian module is Artenian.

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Q.10. If M be a non-zero finite generated module over commutative noetherian ring R. Then show that there are only a finite number of princes associated with M.

OR

State and prove Noether-Lasker theorem.

Q.11. If $\lambda \in F$ be a characteristic root of $T \in A(V)$ then show that λ is a root of the minimal polynomial of T. In particular, T only has a finite number of characteristic roots of F.

OR

Find the Jordan canonical form of A.

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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