

[Maximum Marks : Reg. 85**Pvt.100**

Note :- All questions from each section carry equal marks. All questions are compulsory and answer limit are approximately 250 words. Start the answer of each section from new page. Maximum limit of pages of answer booklet are approximately 16 pages. Answer should be written by the student in his/her own handwriting mandatory. The first page of answersheet should be download by the student from university website www.bubhopal.ac.in is mandatory.

1. If Hilbert-adjoint operator T^* of T exists, then prove that T^* is unique and is a bounded linear operator with norm $\|T^*\| = \|T\|$.
2. State and prove generalized Hahn Banach theorem for complex vector space.
3. Prove that the adjoint operator T^* is linear and bounded, and $\|T^*\| = \|T\|$
4. If a metric space $X \neq \emptyset$ is complete, it is nonmeager in itself. Hence if $X \neq \emptyset$ is complete and $X = \bigcup_{k=1}^{\infty} A_k$ (A_k closed), then prove that at least one A_k contains a nonempty open subset.
5. Let X and Y be Banach spaces and $T : D(T) \rightarrow Y$ a closed linear operator, where $D(T) \subset X$. Then if $D(T)$ is closed in X , then prove that the operator T is bounded.