Total No. of Questions : 11] [Total No. of Printed Pages :6

EF-469

M.A./M.Sc. III[rd] Semester (Reg./Pvt./ ATKT)

Examination, 2021-22

Maths

Paper - XII

Integration Theory-I

Time : 3 Hours] **[Maximum Marks : Reg.= 85, Pvt.= 100**

Note :- Attempt all questions.

SECTION - 'A'

Objective Type Questions 5×3=15

1. Choose the correct answer :

(i) Let (X, M) be a measurable space, If μ and v a[re] measures defined on M, then the set function λ define[d] on M by .... also is a measure.

(a) $\lambda(E) = \mu(E) + v(E)$

(b) $\lambda(E) = \mu(E) - v(E)$

(c) $\lambda(E) = \mu(E) \times v(E)$

(d) None of these

(ii) A decomposition of X into the union of two disjoin[t] sets A and B for which A is positive for v and B negativ[e] is called a Hahn decomposition for ....

(a) X

(b) v

(c) A

(d) None of these

(iii) Let S be a semiring of subsets of a set X. Define S' t[o] be the collection of unions of finite disjoint collection[s] of sets in S. Then S' is .... with respect to th[e] formation of relative complements.

(a) Open

(b) Bounded

(c) Closed

(d) None of these

(iv) The sum of two extended real-valued functions is not defined at points where the functions take infinite value of .... sign.

(a) Same

(b) Positive

(c) Negative

(d) Opposite

(v) Let f and g be nonnegative measureable functions on X for which $g \le f$ a.e. on X, then $f = g$ a. e. on X if and only if ....

(a) $\int_X g\, d\mu = \int_x f\, d\mu$

(b) $\int_X g\, d\mu > \int_x f\, d\mu$

(c) $\int_X g\, d\mu < \int_x f\, d\mu$

(d) None of these

SECTION - 'B'

Short Answer Type Questions 5×5=25

1. Define measurable sets, measurable space and continuity of measure with example.

OR

Let (X, M) be a measureable space. If μ and v are measures on M and $\mu \ge v$ then there is a measure λ on M for which $\mu = v + \lambda$

2. Let v be a singned measure on the measurable space (X, M). Then every measurable subset of a positive set is itself positive and the union of a countable collection of positive sets is positive.

OR

State and prove Hahn's Lemma.

3. Define outer measure and caratbeodory measure with example.

OR

Show that any measure that is induced by an outer measure is complete.

4. Let E be a measureable subset of X and f an extended real-

valued function on X. Show that f is measureable if and only if its restrictions to E and X ~ E are measureable.

OR

Suppose (X, M, μ) is not complete. Show that there is a sequence $\{f_n\}$ of measurable functions on X that converges pointwise a.e. on X to a function f that is not measureable.

5. State and prove Chebychev's Inequality.

OR

State and prove Beppo Levi's Lemma.

SECTION - 'C'

Long Answer Type Questions 9×5=45

1. State and prove Borel - Cantelli Lemma.

OR

Let (X, M) be a measurable space. If μ and v are measures defined on M, then the set function λ defined on M by $\lambda(E) = \mu(E) + v(E)$ also is a measure.

2. State and prove Hahn Decomposition Theorem.

OR

State and prove Jordan Decomposition Theorem.

3. State and prove Caratheodory - Hahn Theorem.

OR

Show that a set function on a σ – algebra is a measure if and only if it is a premeasure.

4. State and prove simple Approximation Theorem.

OR

State and prove Egoroff's Theorem.

5. State and prove Fatou's Lemma.

OR

State and prove the Monotone Convergence Theorem.

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