

10×1.5=15

Roll No.

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EF-442

M.A./M.Sc. Ist Semester (Reg./Pvt./ATKT)

Examination, 2021-22

Maths

Paper - II

Real Analysis

Time : 3 Hours]

**[Maximum Marks : Reg. 85
Pvt. 100]**

Note :- Attempt all the questions.

SECTION - 'A'

Objective Type Questions

1. Choose the correct answers.

(i) The upper Riemann integral of a bounded real function of defined on $[a, b]$ is given by

(a) $\int_a^b f dx = \sup U(P, f)$

(b) $\int_a^b f dx = \inf U(P, f)$

(c) $\int_a^b f dx = \inf U(P, f)$

(d) $\int_a^b f dx = \sup L(P, f)$

(ii) If $f \in R(\alpha)$ on $[a, b]$ and if $|f(x)| \leq M$ on $[a, b]$, then

(a) $\left| \int_a^b f d\alpha \right| \leq M [\alpha(b) - \alpha(a)]$

(b) $\left| \int_a^b f d\alpha \right| \leq M [\alpha(b) + \alpha(a)]$

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(1)

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(2)

(c) $\int_a^b |f| d\alpha \leq M [\alpha(b) + \alpha(a)]$

(d) $\int_a^b |f| d\alpha \leq M [\alpha(a) + \alpha(b)]$

(iii) If a curve γ in R^K is one-to-one then γ is called.

- (a) a closed curve
- (b) an arc
- (c) discontinuous curve
- (d) None of the above

(iv) The length of the curve γ is given by :-

- (a) $\wedge(\gamma) = \inf \wedge(P, \gamma)$
- (b) $\wedge(\gamma) = \sup \wedge(P, \gamma)$
- (c) $\wedge(\gamma) = \wedge(P, \gamma)$
- (d) None of the above

(v) If $\{f_n\}$, $n = 1, 2, 3, \dots$ is a sequence of functions defined on a set E , then $\{f_n\}$ converges to f pointwise on E if:

(a) $f(x) = \lim_{n \rightarrow \infty} f_n(x), (x \in E)$

(b) $f_n(x) = \lim_{n \rightarrow \infty} f(x), (x \in E)$

(c) $f(x) = f_n(x), (x \in E)$

(d) None of the above

(vi) Let $S_{m,n} = \frac{m}{m+n}$, for $m = 1, 2, 3, \dots$ $n = 1, 2, 3, \dots$

then, for every fixed n , $\lim_{m \rightarrow \infty} S_{m,n} = \underline{\hspace{2cm}}$

- (a) 0
- (b) -1
- (c) 1/2
- (d) 1

(vii) A mapping A of a vector space X into a vector space Y is said to be a linear transformation if for all $\vec{x}, \vec{x}_1, \vec{x}_2 \in X$

(a) $A(\vec{x}_1 + \vec{x}_2) = A\vec{x}_1 + A\vec{x}_2$

(b) $A(\vec{x}_1 + \vec{x}_2) = A\vec{x}_1 + A\vec{x}_2$ and $A(c\vec{x}) = cA\vec{x}$
for scalar c .

(c) $A(\vec{x}_1 - \vec{x}_2) = A(\vec{x}_1) + A(\vec{x}_2)$ &

$$A(c\vec{x}) = cA\vec{x}$$

(d) None of the above

(viii) Which of the following statement is false :

- (a) Every span is a vector space
 (b) No independent set contains the null vector.

(c) $|A\vec{x}| \geq \|A\| \cdot |\vec{x}|$

(d) $\{\vec{e}_1, \vec{e}_2, \dots, \vec{e}_n\}$ is called the standard basis of R^n

(ix) Every $A \in L(R^{n+m}, R^n)$ can be split into two linear transformations A_x and A_y defined by for all $\vec{h} \in R^m$ and $\vec{k} \in R^m$

(a) $A_x \vec{h} = A(\vec{h}, \vec{0}), A_y \vec{k} = A(\vec{0}, \vec{k})$

(b) $A_x \vec{h} = A(\vec{0}, \vec{h}), A_y \vec{k} = A(\vec{k}, \vec{0})$

(c) $A_y \vec{h} = A(\vec{h}, \vec{0}), A_x \vec{k} = A(\vec{0}, \vec{k})$

(d) None of the above

(x) The second order partial derivations of a real function defined in an open set ECR^n with partial derivatives $D_1 f, \dots, D_n f$, are defined by :

- (a) $D_i f = D_j f \quad (i, j = 1, 2, \dots, n)$
 (b) $D_{ij} f = D_i D_j f \quad (i, j = 1, 2, \dots, n)$
 (c) $D_j f = D_{ij} f \quad (i, j = 1, 2, \dots, n)$
 (d) $D_i f = D_{ij} f \quad (i, j = 1, 2, \dots, n)$

SECTION - 'B'

Short Answer Type Questions

$5 \times 5 = 25$

2. If $f \in R(\alpha)$ and $g \in R(\alpha)$ on $[a, b]$, then prove that
 $f g \in R(\alpha)$

OR

If $f \in R$ on $[a, b]$ and if there is a differentiable function F on $[a, b]$ such that $F' = f$, then $\int_a^b f(x) dx = F(b) - F(a)$

3. Explain - Rearrangements of terms of a series with an example.

OR

Define a curve γ in R^k . Write a short note on this curve and define its length.

✓ Define the following

(i) Uniform convergence

OR

Define the following.

(i) Point-wise convergence.

✓ Define the following (Any two)

(i) Linear combination,

(ii) Independent set

(iii) Standard basis

OR

Define the following :- (Any two)

(i) Linear Transformation

(ii) Norm of A

(iii) Continuously differentiable function

✓ 6. Define the following :-

(i) Power series

(ii) Derivatives of higher order.

OR

Suppose $\sum c_n$ converges. Put

$$f(x) = \sum_{n=0}^{\infty} C_n x^n \quad (-1 < x < 1) \text{ then prove } \lim_{x \rightarrow 1} f(x) = \sum_{n=0}^{\infty} C_n$$

SECTION - 'C'

Long Answer Type Questions

9×5=45

7. If P^* is a refinement of P , then prove:

$$L(P, f, \alpha) \leq L(P^*, f, d)$$

S

OR

If $f_1 \in R(\alpha)$ and $f_2 \in R(\alpha)$ on $[a, b]$ then prove

(i) $f_1 + f_2 \in R(\alpha)$

(ii) $Cf \in R(\alpha)$ for every constant c .

also prove that

$$\int_a^b (f_1 + f_2) d\alpha = \int_a^b f_1 d\alpha + \int_a^b f_2 d\alpha$$

and $\int_a^b cf d\alpha = C \int_a^b f d\alpha$

8. If f maps $[a, b]$ into \mathbb{R}^k and if $\vec{f} \in R(a)$ for some monotonically increasing function a on $[a, b]$, then $\vec{f} \in R(a)$, and

$$\left| \int f da \right| \leq \int |f| da$$

OR

If γ is continuous on $[a, b]$, then γ is rectifiable, and

$$l(\gamma) = \int |\gamma'(t)| dt$$

9. Suppose $\langle f_n \rangle$ is a sequence of functions, differentiable on $[a, b]$ and such that $\langle f_n(x_0) \rangle$ converges for some point x_0 on $[a, b]$. If $\langle f'_n \rangle$ converges uniformly on $[a, b]$, then $\langle f_n \rangle$ converges uniformly on $[a, b]$, to a function f , and

$$f(x) = \lim_{n \rightarrow \infty} f_n(x) \quad (a \leq x \leq b)$$

OR

(a) If $S_{m,n} = \frac{m}{m+n}$ for $m = 1, 2, 3, \dots$ and $n = 1, 2, 3, \dots$ then prove that $\lim_{m \rightarrow \infty} \lim_{n \rightarrow \infty} S_{m,n} \neq \lim_{n \rightarrow \infty} \lim_{m \rightarrow \infty} S_{m,n}$

(b) State the difference between uniform convergence and point-wise convergence

(c) State cauchy criterion for uniform convergence.

10. Suppose E is an open set in \mathbb{R}^n , \vec{f} maps E into \mathbb{R}^m , \vec{f} is differentiable at $\vec{x}_0 \in E$, \vec{g} maps an open set containing $\vec{f}(\vec{x}_0)$ into \mathbb{R}^k , and \vec{g}' is differentiable at $\vec{f}(\vec{x}_0)$ then the mapping \vec{F} of E into \mathbb{R}^k defined by $\vec{F}(\vec{x}) = \vec{g}\left(\vec{f}(\vec{x})\right)$ is differentiable at \vec{x}_0 and $\vec{F}'(\vec{x}_0) = \vec{g}'\left[\vec{f}(\vec{x}_0)\right]\vec{f}'(\vec{x}_0)$

OR

(a) Define partial derivatives

(b) Suppose \vec{f} maps a convex open set $E \subset \mathbb{R}^n$ into \mathbb{R}^m , \vec{f} is differentiable in E , and there is a real number M such that

$$\|\vec{f}'(x)\| \leq M \text{ for every } \vec{x} \in E \text{ then}$$

$$f(b) - f(a) \leq M \left| b - a \right|$$

for all $a \in E, b \in E$

11. Suppose the series $\sum_{n=0}^{\infty} C_n x^n$ converges for $|x| < R$, and define

$$f(x) = \sum_{n=0}^{\infty} C_n x^n \quad \text{for } |x| < R. \quad \text{Prove that the series } \sum_{n=0}^{\infty} C_n x^n$$

converges uniformly on $[-R + \epsilon, R - \epsilon]$ no matter which $\epsilon > 0$ is chosen.

OR

Suppose $f(x) = \sum_{n=0}^{\infty} C_n x^n$ this series converges in $|x| < R$. If

$R > a > R$, then f can be expanded in a power series about the point $x = a$ which converges in $|x - a| < R - a$, and

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x - a)^n, \quad (x - a < R - a)$$

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