

SV-402

M.A./M.Sc. 1st Semester (Reg./Pvt.) (NEW/ATKT)

Mathematics Examination December, 2017

COMPLEX ANALYSIS - I

Paper - IV

Time Allowed : Three Hours] [Maximum Marks : { Reg. - 85
Pvt. - 100

Note : Attempt all the questions.

Section - A

(Objective Type Questions)

Q.1. Choose the correct answers: 5x2=10

- i) The value of $\int_C \frac{dz}{z}$, where C is the circle with centre at the origin and radius r,
 - (a) π
 - (b) πi
 - (c) $2\pi i$
 - (d) None of these
- ii) If $f(z)$ is analytic at each point in some nbd $|z-a| < R$ of a except at the point a itself. Then point 'a' is said to be:
 - (a) Removable singularity
 - (b) Isolated singularity
 - (c) Essential singularity
 - (d) None of these
- iii) Let $f(z) = \frac{1}{(z-1)^2(z-3)^5}$, then $z=3$ is a pole of order:
 - (a) 2
 - (b) 3
 - (c) 6
 - (d) 8

- iv) If a single valued function has only a finite number of singularities, then the sum of residues at these singularities and including the residue at infinity is
 - (a) 0
 - (b) 1
 - (c) 2
 - (d) ∞
- v) The fixed point of the bilinear transformation $w = \frac{z-1}{z+1}$ is:
 - (a) $-i$
 - (b) i
 - (c) Both
 - (d) None of these

Section - B

(Short Answer Type Questions)

5x5=25

Q.2. Find the value of the integral $\int_0^{1+i} (x-y+ix^2) dx$.

OR

Evaluate the following integral $\int_L |dz|$ where L is any rectifiable arc joining the point $z = \alpha$ and $z = \beta$.

Q.3. Expand $\log(1+z)$ in a Taylor's series about $z=0$ and determine the region of convergence for the resulting series.

OR

- a) Define the entire function
- b) Write the statement of fundamental theorem of algebra.

Q.4. Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in Laurent series valid for

- a) $|z| > 3$
- b) $0 < |z+1| < 2$

OR

(3)

Find the kind of singularity of the function

$$f(z) = \frac{\cot \pi z}{(z-a)^2} \text{ at } z=a \text{ and } z = \infty$$

Q.5. State and prove that Cauchy's residue theorem.

OR

Find the residue of the function $f(z) = \frac{1}{(z^2 + a^2)^2}$ at $z = ai$.

Q.6. Find the bilinear transformation which maps $z_1 = 2, z_2 = i, z_3 = -2$ at the points $w_1 = 1, w_2 = i$ and $w_3 = -1$.

OR

Define the following:

- a) Crossed Ratio
- b) Fixed Point

Section - C

(Long Answer Type Questions)

5x10=50

Q.7. State and prove Cauchy Goursat theorem.

OR

If $f(z)$ be analytic within or on the closed contour C and let a be any point within C , then prove that

$$f'(a) = \frac{1}{2\pi i} \int_C \frac{f(z)}{(z-a)^2} dz$$

(4)

Q.8. State and prove Liouville's theorem.

OR

State and prove Morera's theorem.

Q.9. State and prove Maximum modulus principle.

OR

Prove that all the roots of $z^5 - 5z^3 + 12 = 0$ lie between the circles $|z| = 1$ and $|z| = 2$.

Q.10. Prove that $\int_0^{2\pi} e^{-\cos \theta} \cos(n\theta + \sin \theta) d\theta = \frac{2\pi(-1)^n}{n!}$ where n is a integer.

OR

Applying the calculus of residue to prove that

$$\int_{-\infty}^{\infty} \frac{x^2}{1+x^4} dx = \frac{\pi}{\sqrt{2}}$$

Q.11. Find the bilinear transformation which maps the circle into the circle $|z-1| = 1$ and map $w=0, w=1$ into $z=0$ respectively. Also show that the transformation is uniquely determined.

OR

Prove that the set of all bilinear transformation forms a non-abelian group under the product of transformation.

