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SV-399

M.A./M.Sc. 1st Semester (Reg./Pvt.) (NEW/ATKT) Mathematics Examination December, 2017

ADVANCE ABSTRACT ALGEBRA-I

Paper - I

Time Allowed : Three Hours] *[Maximum Marks : { Reg. - 85, Pvt. - 100*

Note : Attempt all the questions from Section "A", "B" and "C". Internal choice is given where necessary. Marks are indicated. Mathematical notations are in their usual meanings.

Section - A
(Objective Type Questions)

Q.1. Choose the correct answer: 10×2=20

i) If any group G has no proper normal subgroup then G is
(a) Simple (b) Perfect
(c) Cyclic (d) None of these

ii) Every finite group has a
(a) Decomposition series (b) Composition series
(c) Subnormal series (d) None of these

iii) Every abelian group is
(a) Simple (b) Cyclic
(c) Solvable (d) None of these

iv) If a group G has a normal series, then G is
(a) Nilpotent (b) Cyclic
(c) Simple (d) Perfect

v) Finite extension of a finite field is always
(a) Inseparable (b) Separable
(c) Perfect field (d) None of these

vi) R is an extension field of Q, then for the real number π which of the following is true
(a) π is algebraic over Q
(b) π is transcendental over R
(c) π is algebraic over R
(d) None of these

vii) If F is a field such that each of its algebraic extension is separable, then F is a
(a) Perfect field (b) Simple field
(c) Finite field (d) None of these

viii) Every finite field is a
(a) Prime field
(b) Perfect field
(c) Both prime and perfect field
(d) None of these

ix) Any extension E of a field F, is a Galois extension of F, if:
(a) E is only finite extension of F
(b) E is only finite and normal extension of F
(c) E is finite, normal and also separable extension of F
(d) None of these

x) The set of all automorphisms Aut(F) a field F forms
(a) A group (b) A ring
(c) A field (d) An Integral domain
Under compositions of mappings

Section - B
(Short Answer Type Questions)

5×5=25

Q.2. Define subnormal and normal series.

OR

Give an example of a composition series of a group and explain it.

Q.3. Define solvable group with example.

OR

Show that every homomorphic image of a solvable group is solvable.

Q.4. Define splitting field with example.

OR

Show that every finite extension of a field is an algebraic extension..

Q.5. Define prime field with example. What is Galois field?

OR

Discuss about algebraically closed fields and algebraic closure of a field.

Q.6. Show that the set of all automorphisms on a field F means Aut (F) forms a group into composition of mappings.

OR

Show that the general polynomial of degree $n \geq 5$ is not solvable by radicals.

Section - C

(Long Answer Type Questions)

5×8=40

Q.7. Show that, if G is a commutative group having a composition series then G is finite.

OR

Show that any two composition series of a finite group are equivalent.

Q.8. Prove that if H is a normal subgroup of a solvable group G, then G/H is also solvable.

OR

Show that every subgroup and every homomorphic image, of a nilpotent group is nilpotent.

Q.9. Let $F \subseteq E \subseteq K$ be field. If K is a finite extension of E and E is a finite extension of F, then K is a finite extension of F and $[K:F]=[K:E]\,[E:F]$

OR

Show that every finite separable extension of a field is necessarily a simple extension.

Q.10. Show that, if F is a field then there exists an algebraically closed field K containing F as subfield.

OR

Let E be a finite extension of a field F. Then the following are equivalent:

a) $E=F(\alpha)$ for some $\alpha \in F$

b) There are only a finite number of intermediate fields between F and E.

Q.11. Let E be a finite separable extension of a field F and $H<G(E/F)$, then $G(E/E_H)=H$ and $[E:E_H]=|G(E/E_H)|$

OR

Show that the cubic polynomial:

$f(x)=x^3+3x^2+3bx=c\in Q(x)$ over Q is solvable by radicals.

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