Roll No.: ....

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SV-427

M.A./M.Sc. 3rd Semester (Reg./Pvt.) (NEW/ATKT) Mathematics Examination Nov./Dec., 2017

INTEGRATION THEORY-I

Optional Paper selct any four

Paper - XII

Time Allowed : Three Hours] [Maximum Marks : { Reg. - 85, Pvt. - 100

Note : Attempt all questions.

Section - 'A'

Objective Type Questions

5x2=10

Q.1. Fill in the blanks.

i) If $m^*E = 0$, then E is __________ .

ii) Every measurable subset of a positive set is _______ positive.

iii) The set function μ^* is an ________ measure.

iv) Let C be a constant function and *f* and *g* are two _____ real valued functions defined on the same domain. Then f+c and *g*–*f* are also measurable.

v) A non-negative measurable function f is called ______ over the measurable set E if

$$\int_E f < \infty$$

Section - 'B'

Short Answer Type Questions

 5x5=25

Q.2. Let E_1 and E_2 are measurable, then prove that $E_1 \cup E_2$ is measurable.

OR

Define finite and σ - finite measure of a set.

Q.3. What do you understand by a signed measure on the measurable space (X, B).

OR

Prove that the union of a countable collection of positive sets is positive.

Q.4. Define outer measure μ^*. Describe when a set is said to be measurable with respect to μ^*.

OR

What do you understand by a measure on an algebra.

Q.5. If *f* is a measurable function and f = g almost everywhere, then prove that *g* is also measurable.

OR

Let f be an extended real valued function defined on X. Then prove that the following statements are equivalent :

i) $\{x : f(x) < \alpha\} \in B$ for each α.

ii) $\{x : f(x) > \alpha\} \in B$ for each α.

Q.6. Let f and g be non-negative measurable functions and if $f \le g$ a.e., then prove that

$$\int_E f \le \int_E g$$

OR

Let f and g be two non-negative measurable functions. If f is integrable over E and $g(x) < f(x)$ on E, then prove that g is also integrable on E and

$$\int_E f - g = \int_E f - \int_E g$$

Section - 'C'

Long Answer Type Questions

5×10=50

Q.7. Prove that the interval (a,∞) is measurable.

OR

Let (X, B, μ) be a measure space. If $E_i \in B$, then prove that

$$\mu\left(\bigcup_{i=1}^{\infty} E_i\right) \le \sum_{i=1}^{\infty} \mu E_i$$

Q.8. State and prove Hahn Decomposition Theorem.

OR

Let E be a measurable set such that $0 < \nu E < \infty$. Then prove that there is a positive set A contained in E with $\nu A > 0$.

Q.9. Prove that the class B of μ^* measurable sets is a σ-algebra and if $\bar{\mu}$ is μ^* restricted to B, then $\bar{\mu}$ is a complete measure on B.

OR

State and prove Caratheodory Hahn Theorem.

Q.10. State and prove Simple Approximation Theorem.

OR

State and prove Egoroff's Theorem.

Q.11. State and prove monotone convergence Theorem.

OR

Let f be a non-negative function which is integrable over a set E. Then prove that for a given $\epsilon > 0$ there is a $\delta > 0$ such that for every set $A \subset E$ with $mA < \delta$,

$$\int_A f < \epsilon$$

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