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EG-121

B.E. I Semester (CGPA) Mech. Engg. Examination 2018

ENGINEERING MATHEMATICS - I

Paper - M-101

Time Allowed : Three Hours] *[Maximum Marks : 60*

Note : Attempt all questions. All Questions carry equal marks.

Q.1. a) Write the statement of mean value theorem.

b) What is the value of $\sqrt{\left(\frac{1}{3}\right)} \times \sqrt{\left(\frac{2}{3}\right)}$.

c) The value of $\lim_{n\to\infty}\left[\frac{1}{n+1}+\frac{1}{n+2}+---\frac{1}{2n}\right]$ is ___.

d) Write the equation of tangent at the point (x,y).

e) Evaluate $\int_0^{\pi/2} \cos^3 x dx$.

Q.2. a) Expand by Maclaurin's theorem $e^{x\cos x}$ as for as the term x^3.

b) The radius of a sphere is found to be 10 cm with a possible error of 0.02 cm. What is the relative error in computing the volume?

OR

c) Verify mean value theorem for the function $f(x) = 2x^2 - 10x + 29 in [2,7]$.

d) Prove that

$$\log \sec x = \frac{1}{2}x^2 + \frac{1}{12}x^4 + \frac{1}{45}x^6 + \ldots$$

Q.3. a) Find the equations to the tangent and normal at the point $(a\cos\theta, b\sin\theta)$ to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

b) Find the radius of curvature at the given point

$\sqrt{x} + \sqrt{y} = \sqrt{a}$ at $\left(\frac{1}{4}, \frac{1}{4}\right)$.

OR

c) Find the radius of curvature at any point t of the curve
$x = 3a\cos t - a\cos 3t$.
$y = 3a\sin t - a\sin 3t$

d) Find the coordinate of the centre of curvature for any point (x,y) on the parabola $y^2 = 4ax$.

Q.4. a) If $U = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right)$ show that

$$x\frac{\partial U}{\partial x} + y\frac{\partial U}{\partial Y} = 0.$$

b) Discuss the maxima and minima of the function $U = xy + \frac{a^3}{x} + \frac{a^3}{y}$.

OR

c) find the maxima or minima of the function

$$U = 2\sin\frac{x+y}{2}.\cos\frac{x-y}{2} + \cos(x+y)$$

d) If $U = x^4y^2\sin^{-1}\left(\frac{y}{x}\right)$ then prove that

$$x\frac{\partial U}{\partial x} + y\frac{\partial U}{\partial Y} = 6U$$

Q.5. a) Evaluate from limit of sum $\int_a^b x\,dx$.

b) Find the volume bounded by the paraboloid $x^2 + 4y^2 + z = 4$ and xy- plane.

OR

c) Find the area bounded by y. axis the line $y = 2x$ and the line $y = 4$.

d) Evaluate

$$\lim_{n\to\infty}\left[\frac{n}{n^2} + \frac{n}{n^2+1} + \frac{n}{n^2+2^2} + \ldots + \frac{n}{n^2+(n+1)^2}\right]$$

Q.6. a) Prove that $\int_0^1 x^2(1-x)^3\,dx = \frac{1}{60}$.

b) Evaluate $\int_0^a \int_0^{\sqrt{a^2-y^2}} \sqrt{a^2-x^2-y^2}\,dx\,dy$.

OR

c) Prove that $\int_0^\infty \frac{e^{-st}}{\sqrt{t}}dt = \sqrt{\frac{\pi}{s}}.s > 0$.

d) Evaluate $\iint_R e^{2x+3y}\,dx\,dy$ over the triangle bounded by $x = 0$, $y = 0$ and $x + y = 1$.

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