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M.A./M.Sc. 3rd Semester Special (ATKT) Examination July, 2017

SP–156

MATHEMATICS
(Integral Transform–I)
(Optional Paper)
Paper : X

Time : 3 Hours] [Maximum Marks : {Reg. = 85, Pvt. =100

Note :– Attempt all questions.

Section–A

(Objective Type Questions) 5×2=10

1. Choose the correct answer.

(i) L {sin *at*} is equal to :

(a) $\frac{p}{p^2+a^2}$

(b) $\frac{p^2}{p^2+a^2}$

(c) $\frac{a^2}{p^2+a^2}$

(d) $\frac{a}{p^2+a^2}$

(ii) If $y = y(x, t)$, then $L\left\{\frac{dy}{dx}\right\}$ is equal to :

(a) $x\bar{y}(x,s)+y(x,0)$

(b) $s\bar{y}(x,s)-y(x,0)$

(c) $s\bar{y}(x,0)-y(x,s)$

(d) $\bar{y}(s)$

(iii) One dimensional wave equation is given by :

(a) $\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}=0$

(b) $\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$

(c) $\frac{\partial^2 v}{\partial t^2} = c^2 \frac{\partial^2 v}{\partial x^2}$

(d) $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$

(iv) If ends of a beam are simply supported ends the condition is :

(a) $y'' = y''' = 0$

(b) $y = y'' = 0$

(c) $y = y' = 0$

(d) $y = 0$

(v) In one dimensional heat conduction equation

$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}, k$ is stand for :

(a) $\frac{K}{C\rho}$

(b) EI

(c) I = QC

(d) ρ

Section–B

(Short Answer Type Questions) 5×6=30

2. Find :

$$L^{-1}\left\{\frac{s-2}{(s-2)^2+5^2} + \frac{5+4}{(s+4)^2+9^3} + \frac{1}{(s+2)^2+3^2}\right\}$$

Or

Write the statement of convolution theorem.

3. Using Laplace transform, find the solution of $y'' + 25y = 10 \cos (5t)$, where $y(0) = 2$, $y'(0) = 0$.

Or

Solve $F'(t) = t + \int_0^t F(t-u)\cos u\,du$, F (0) = 4.

4. Find the solution of two dimensional Laplace's equation in Cartesian co-ordinates by the method of separation of variables.

Or

A tightly strectched string fixed end point $x = 0$ and $x = l$ is initially in a position given by :

$$y(x,0) = y_0 \sin\frac{\pi x}{l}$$

If it is released from rest from this position, find the displacement y at any distance x from one end at any time t.

5. An inductor of 2 Henrys, a resistance of 16 ohm and a capacitor of 0.02 Farads are connected in series with an e.m.f. of 100 sin $3t$ volts. At $t = 0$ the charge on the capacitor and current in the circuit are zero. Find the charge and current at any time $t > 0$.

Or

A beam which is clamped at its ends $x = 0$, $x = l$ carries a uniform load W_0 per unit length. Show that the deflection at any point is :

$$y(x) = \frac{w_0 x^2 (l-x)^2}{24EI}$$

6. Write the equation for one dimensional flow of heat along a bar which is in the form of :

$$\frac{\partial u}{\partial t} = h^2 \frac{\partial^2 u}{\partial x^2}$$

Or

Determine the solution of one dimensional heat equation $\frac{\partial \theta}{\partial t} = a^2 \frac{\partial^2 \theta}{\partial x^2}$ under the boundary conditions $\theta(0, t) = 0$, $\theta(l, t) = 0$, $t > 0$ and the initial condition $\theta(x, 0) = x$, $0 < x < l$, l being the length of the bar.

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Section–C

(Long Answer Type Questions) 5×9=45

7. Find the Laplace transform of $\sin\left(\sqrt{t}\right)$.

Or

Find the inverse Laplace transform of $\frac{3p+7}{p^2-2p-3}$.

8. Solve the differential equation :

$$ty''(t) + y'(t) + ty(t) = 0$$

under the conditions that $y(0) = 1$ and $y(t)$ and its derivatives have transforms.

Or

Solve :

$$(D - 2)\, x - (D + 1)\, y = 6\, e^{3t}$$
$$(2D - 3)\, x + (D - 3)\, y = 6\, e^{3t}$$
$$x(0) = 3,\ y(0) = 0$$

9. To find out the solution of :

$$\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2}$$

by the method of separation of variables.

Or

A rectangular plate with insulated surfaces is 10 cm. wide and so long compared to its width that it may be considered infinite in length without introducing an appreciable error. If the temperature along the short edge $y = 0$ is given by :

$$u(x, 0) = 20x, \ 0 \leq x \leq 5$$

$$u(x, 0) = 20(1 - x), \ 5 < x < 10$$

while the two long edges $x = 0$ and $x = 10$ as well as the other short edge are kept at 0°C. Find the steady state temperature a point (x, y) of the plate.

10. An alternating E.M.F. E sin wt is applied to an inductance and a capacitance C in series. Show that the current in the circuit is :

$$\frac{E\omega}{(n^2 - \omega^2)L}(\cos \omega t - \cos nt)$$

where $n^2 = 1/EC$.

Or

A beam which is hinged at its ends $x = 0$ and $x = l$ carries a uniform load W_0 per unit length. Find the deflection at any point.

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11. A semi-infinite solid $x > 0$ initially at temperature zero. At time $t = 0$, a constant temperature $u_0 > 0$ is applied and maintained at the face $x = 0$. Find the temperature at any point of the solid at any time $t > 0$.

Or

A semi-infinite solid $x > 0$ has its initial temperature equal to zero. A constant heat flux A is applied at the face $x = 0$. Find the temperature at any point $x > 0$ of the solid given that :

$$L^{-1}\left\{\frac{e^{-x\sqrt{s}}}{s^{3/2}}\right\} = e^{-x^2/4t^2}\left(\frac{t}{\pi}\right)^{\frac{1}{2}} - \frac{x}{2}erfc\left(\frac{x}{2\sqrt{t}}\right)$$

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